65. $\frac{dy}{dx} = x - \frac{1}{x^2}$ $\quad \frac{1}{x^2} = x^{-2}$

a) $(0,\infty)$ $y(1)=2$

$y = \frac{x^2}{2} + \frac{1}{x} + c$

$2 = \frac{1}{2} + \frac{1}{1} + c$

$c = \frac{1}{2}$

$y = \frac{x^2}{2} + \frac{1}{x} + \frac{1}{2}$

b) $(-\infty,0)$ $y(-1)=1$

$y = \frac{x^2}{2} + \frac{1}{x} + c$

$1 = \frac{1}{2} - 1 + c$

$c = \frac{3}{2}$

$y = \frac{x^2}{2} + \frac{1}{x} + \frac{3}{2}$

c) $y = \begin{cases} \frac{1}{x} + \frac{x^2}{2} + c_1, & x<0 \\ \frac{1}{x} + \frac{x^2}{2} + c_2, & x>0 \end{cases}$

show this is a solution to $\frac{dy}{dx} = x - \frac{1}{x^2}$

$\frac{dy}{dx} = \begin{cases} -\frac{1}{x^2} + x, & x<0 \\ -\frac{1}{x^2} + x, & x>0 \end{cases}$

Dec 1-9:21 AM

55. $\frac{dy}{dx} = e^{\frac{x-y}{2}}$

$\perp$

$\frac{dy}{dx} = -e^{\frac{y-x}{2}}$

neg. recip.: $-\frac{1}{e^{\frac{x-y}{2}}} = -e^{-\left(\frac{x-y}{2}\right)}$

Dec 1-9:43 AM

6.2a Integration by Substitution

A change of variables can turn an unfamiliar integral into one that we can evaluate. (The differential matters.)

$$\int f(x)\,dx = \int g(u)\,du$$

hard easy

inside of the composite

$$\int \sin(x) e^{\cos(x)}\,dx$$

let $u = \cos x$

$$\frac{du}{dx} = -\sin x$$

$$du = -\sin x \cdot dx$$

$$dx = \frac{du}{-\sin x}$$

$$\int \cancel{\sin(x)}\, e^u \frac{du}{-\cancel{\sin x}}$$

$$-\int e^u\,du = -e^u + c$$

$$= -e^{\cos x} + c$$

Nov 30-6:31 PM

$$\int x^2\sqrt{5+2x^3}\,dx \quad u = 5+2x^3$$

$$\frac{du}{dx} = 6x^2$$

$$du = 6x^2\,dx$$

$$\frac{du}{6x^2} = dx$$

$$\int \cancel{x^2}\sqrt{u}\,\frac{du}{6\cancel{x^2}}$$

$$\frac{1}{6}\int \sqrt{u}\,du$$

$$\frac{1}{6}\int u^{1/2}\,du = \frac{1}{6}\,\frac{u^{3/2}}{3/2} + c$$

$$= \frac{1}{6}\cdot\frac{2}{3}\left(5+2x^3\right)^{3/2} + c$$

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$$\int \cot(7x)\,dx$$

$$\int \frac{\cos(7x)}{\sin(7x)}\,dx \qquad \text{let } u = \sin(7x)$$

$$du = 7\cos(7x)\cdot dx$$

$$\int \frac{\cancel{\cos(7x)}}{u}\,\frac{du}{7\cancel{\cos(7x)}} \qquad \frac{du}{7\cos(7x)} = dx$$

$$\frac{1}{7}\int \frac{1}{u}\,du = \frac{1}{7}\ln|u| + c$$

$$= \frac{1}{7}\ln|\sin(7x)| + c$$

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$$\int \frac{dx}{\cos^2 2x} = \int \sec^2(2x)\,dx$$

$$= \frac{1}{2}\tan(2x) + c$$

$u = 2x$

$du = 2dx$

$dx = \frac{du}{2}$

$$\int \sec^2(u)\,\frac{du}{2} = \frac{1}{2}\int \sec^2 u\,du$$

$$= \frac{1}{2}\tan u + c$$

$$= \frac{1}{2}\tan(2x) + c$$

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$\int \cot^2 3x\, dx$

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$\int \cos^3 x\, dx = \int \cos^2 x \cos x\, dx$

$= \int (1-\sin^2 x)\cos x\, dx$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1-\sin^2 x$

let $u = \sin x$

$du = \cos x\, dx$

$= \int 1-u^2\, du = u - \frac{u^3}{3} + C$

$= \sin x - \frac{\sin^3 x}{3} + C$

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Definite Integrals

$$\int_0^{\frac{\pi}{3}} \tan x \sec^2(x)\,dx$$

$u = \tan x$

$du = \sec^2 x\,dx$

$$\int_0^{\sqrt{3}} u\,du$$

if $x = 0$ then $u = \tan 0 = 0$

if $x = \frac{\pi}{3}$ then $u = \tan\frac{\pi}{3}$

$u = \sqrt{3}$

$$\left.\frac{u^2}{2}\right|_0^{\sqrt{3}}$$

$$\frac{3}{2} - 0 = \frac{3}{2}$$

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$$\int_0^1 \frac{x}{x^2-4}\,dx$$

Nov 30-7:00 PM